

Exercise 41

Find the critical numbers of the function.

$$f(\theta) = 2 \cos \theta + \sin^2 \theta$$

Solution

A critical number is a value of θ for which the derivative is zero or nonexistent. Take the derivative of the function.

$$\begin{aligned} f'(\theta) &= \frac{d}{d\theta}(2 \cos \theta + \sin^2 \theta) \\ &= -2 \sin \theta + 2 \sin^1 \theta \cdot \frac{d}{d\theta}(\sin \theta) \\ &= -2 \sin \theta + 2 \sin \theta \cos \theta \\ &= 2 \sin \theta(-1 + \cos \theta) \end{aligned}$$

Set $f'(\theta) = 0$ and solve for θ .

$$2 \sin \theta(-1 + \cos \theta) = 0$$

$$\sin \theta = 0 \quad \text{or} \quad \cos \theta = 1$$

$$\theta = 0 + n\pi \quad \text{or} \quad \theta = 0 + 2n\pi, \quad n = 0, \pm 1, \pm 2, \dots$$

The critical numbers are $0, \pm\pi, \pm 2\pi, \dots$